Time-frequency Analysis of Variable Star Light Curves - The Program Package TiFrAn

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Abstract

We present the properties of the new software, TiFrAn developed for scientific applications of time-frequency analysis. We implemented the general (Cohen's type) kernel method for calculating time-frequency distributions in our program. We show the efficiency and usefullnes of this package by means of analysing semi-regular variable star light curves.
Keywords: *variable stars, time-frequency analysis*

1. Introduction

The development of the TiFrAn (Time-Frequency Analyser) program package was started in 2001. Our goal was to build a program for examination of time-dependent periodic and quasi-periodic processes. TiFrAn can perform several time-frequency methods. An additional aim was to extend the analysing power of the code with a data acquisition system for real-time signal processing.

The major part of the program (without the data acquisition system) is available freely for scientific and educational purposes, and it is running on UNIX based and MS Windows operating systems.[1]

TiFrAn already has been used in many scientific applications successfully. Several semi-regular variable star light curves were analysed with its help (Buchler&Kolláth, 2001).

2. Properties of TiFrAn

2.1. Main components

- Software engine (C): This engine performs the data processing, the calculation of several time-frequency distributions (for example STFT, Gábor, Wavelet, Wigner-Ville, Choi-Williams, Zhao-Atlas-Marks, see 2.2.), and

[1] See the web site *http://www.konkoly.hu/tifran.*

provides additional functionalities (Fourier-transformation, interpolation, filtering, whitening).

- Programmable Graphic User Interface (extended Tcl/Tk): With this simple language, the user can easily define a new graphical interface for special purposes. Menus, buttons, plot fields can be easily created. Several example GUIs are provided with the package.
- Extended Tcl script language: Our script language is suitable for definition of complex (and repeatable) tasks, and creating high-quality Postscript figures.
- Data acquisition system: The full version of the software includes sound card and data acquisition card support, and real-time data processing routines.

2.2. Time-frequency methods

We used the Cohen type kernel method for calculating time-frequency distributions. Different type of distributions can be defined with this formalism, which can be written in the following form:

$$C(t,\omega) = \frac{1}{4\Pi^2} \iiint s^*(u-\frac{\tau}{2})s(u+\frac{\tau}{2})\Phi(\theta,\tau)e^{-j\theta t - j\tau\omega + j\theta u} du d\tau d\theta \quad (1)$$

where s is the signal and $\Phi(\theta,\tau)$ is the kernel function (Cohen, 1995). In the case of $\Phi(\theta,\tau)=1$, Eq (1) gives the Wigner-Ville distribution (Wigner, 1932), (Ville, 1948). The main problem with the Wigner-Ville distribution is the appearance of cross terms in multi-periodic signals. By using a suitable kernel the cross-terms are efficiently reducible but at the cost of some loss in resolution, so we have to make a compromise at choosing the correct kernel. Choice of the optimal kernel is a hard problem in many applications. The kernel functions of Wigner-Ville, Choi-Williams (Choi&Williams, 1989) and Zhao-Atlas-Marks (Zhao et al, 1990) distributions can be found in Table 1.

3. Light curve analysis

3.1. S Draconis - Comparison of time-frequency methods

In Fig 1 we show the light curve of the semi-regular variable star S Draconis and the time-frequency representations calculated by three different methods.

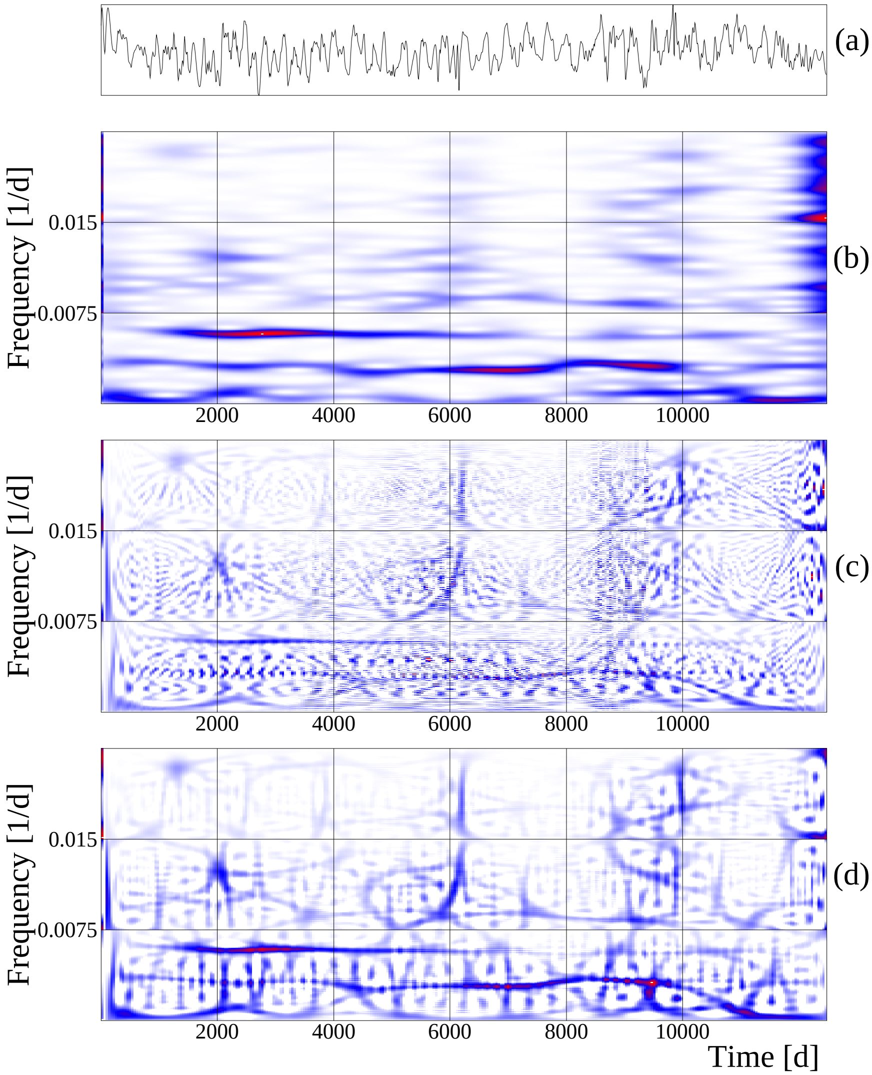

Figure 1: Light curve (a), STFT (b), Wigner-Ville (c) and Choi-Williams (d) distribution of semi-regular variable star S Draconis.

Table 1: Sample kernels

Kernel	$\Phi(\theta, \tau)$
Wigner-Ville	1
Choi-Williams	$e^{-\theta^2\tau^2/\sigma}$
Zhao-Atlas-Marks	$e^{-\alpha\tau^2 \frac{\sin\theta\tau/2}{\theta\tau/2}}$

Components at high frequencies are much weaker than at low frequencies, so we used frequency filtering during the calculations for amplifying and visualizing these components. Low-frequency parts of the distributions were constructed with filtering out the high-frequency domain of the signal and vice versa. Bounds of the filtering are represented by horizontal lines in the figures.

STFT (Short Time Fourier Transformation) distribution can be seen below the light curve. The disadvantage of this method is the relatively low resolution. Wigner-Ville method gives a high resolution, but strong cross-terms appear between the different signal components. By using an appropriate kernel most of these cross-terms can be removed. The result with the Choi-Williams kernel is shown on the bottom of Fig (1). With the Zhao-Atlas-Marks kernel we obtained very similar distribution to the Choi-Williams one.

The fundamental mode of S Draconis can be easily found at the frequency of $f \approx 0.007\ 1/d$. At $t \approx 6000d$ this component weakens and at the same time a component with a lower frequency is rising. The star seems to change its base frequency. After $10000d$ the frequency of this new component decreases.

From time to time components with varying frequencies appear in the higher frequency domain. These outbursts can be observed simultaneously at several frequencies.

3.2. X Herculis and V Bootis

X Her and B Boo are also semi-regular variables. The Choi-Williams distributions of these stars are presented in Fig 2. We used the above mentioned filtering to amplify the low amplitude parts of the signal.

High amplitude modulation can be detected in the main period of X Her ($f \approx 0.01$). A low frequency component with a strongly varying frequency and amplitude also exists in the light curve. The amplitude variation of these two components are in opposite phase, alluding to the interaction between them.

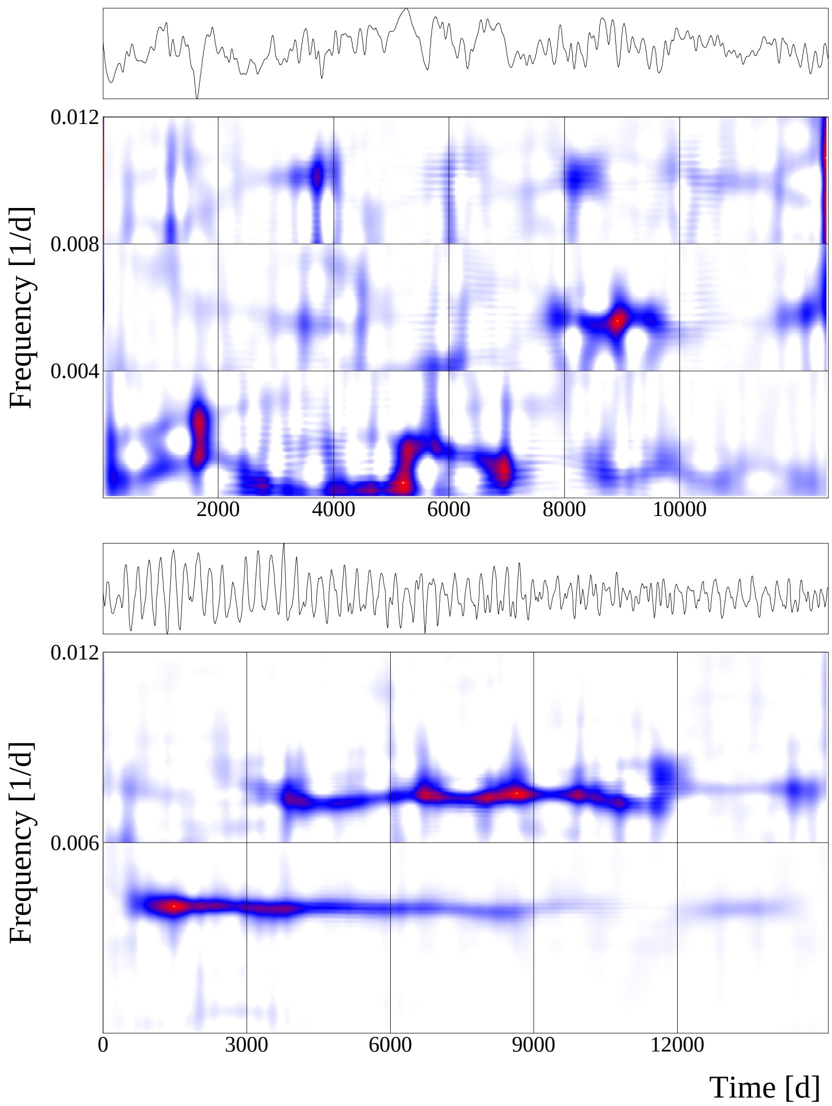

Figure 2: Light curve and Choi-Williams distribution of two semi-regular variable stars: X Herculis (top) and V Bootis (bottom).

The main frequency component ($f \approx 0.04\ 1/d$) of V Bootis is weakening continuously (this effect can also be seen in the light curve), while a higher frequency component ($f \approx 0.08\ 1/d$) is rising (note that this component was amplified artifically by filtering, so it is still lower in amplitude than the main component). Both of these modes are amplitude and frequency modulated, which modulations occur almost in phase.

4. Conclusion

Modern time-frequency methods can be used successfully in the analysis of variable star light curves. We can find new features in the data which cannot be detected with traditional methods. New results may be expected by analysing even well-known, well-examined light curves. Amplitude and frequency evolutions, interactions between modes become evident in time-frequency representations.

Acknowledgements

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