

## LONG-PERIOD VARIABILITY IN *o* CETI

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Received 2007 December 20; accepted 2008 October 12; published 2009 February 3

### ABSTRACT

We carried out a new and sensitive search for long-period variability in the prototype of the Mira class of long-period pulsating variables, *o* Ceti (Mira A), the closest and brightest Mira variable. We conducted this search using an unbroken light curve from 1902 to the present, assembled from the visual data archives of five major variable star observing organizations from around the world. We applied several time-series analysis techniques to search for two specific kinds of variability: long secondary periods (LSPs) longer than the dominant pulsation period of  $\sim 333$  days, and long-term period variation in the dominant pulsation period itself. The data quality is sufficient to detect coherent periodic variations with photometric amplitudes of 0.05 mag or less. We do not find evidence for coherent LSPs in *o* Ceti to a limit of 0.1 mag, where the amplitude limit is set by intrinsic, stochastic, low-frequency variability of approximately 0.1 mag. We marginally detect a slight modulation of the pulsation period similar in timescale to that observed in the Miras with meandering periods, but with a much lower period amplitude of  $\pm 2$  days. However, we do find clear evidence of a low-frequency power-law component in the Fourier spectrum of *o* Ceti's long-term light curve. The amplitude of this stochastic variability is approximately 0.1 mag at a period of 1000 days, and it exhibits a turnover for periods longer than this. This spectrum is similar to the red noise spectra observed in red supergiants.

**Key words:** convection – stars: AGB and post-AGB – stars: individual (*o* Ceti, Mira) – stars: oscillations – stars: variables: other

### 1. INTRODUCTION

The bright variable *o* Ceti (Mira A) is the class prototype of the *Mira variables*, which are pulsating asymptotic giant branch (AGB) stars with periods of hundreds of days, and visual light amplitudes of between 2.5 and 10 mag. *o* Ceti, discovered in 1596, is also the brightest member of its class, with visual maxima routinely reaching 3rd magnitude. *o* Ceti is also one of the closest Mira variables to us, and because of its large size (over  $500R_{\odot}$ ) was resolved with the *Hubble Space Telescope* and interferometric techniques (e.g., Karovska et al. 1991, 1997; Wilson et al. 1992; Haniff et al. 1995; Woodruff et al. 2008). Because of its brightness, its proximity, and its long observational history, *o* Ceti provides an excellent laboratory for studying the Mira variables generally. *o* Ceti is also a symbiotic binary, as it has a companion—VZ Ceti (Mira B)—at a distance of approximately 70 AU (e.g., Karovska 1992; Karovska et al. 2005; Ireland et al. 2007) which is accreting mass from Mira A's substantial wind.

Pulsating AGB stars generally show very irregular pulsation behavior. The Mira variables are an exception to the rule among AGB stars, but even so they are generally not stable in period at a level of less than 1% from cycle to cycle, and their maxima, minima, and light curve shape often vary significantly from cycle to cycle as well. A small fraction have large excursions in period, amplitude, and mean light over the course of their observational histories, the causes of which are not known. A small fraction ( $\sim 10\%$ ) exhibit clearly measurable period variations over timescales of decades; these stars were dubbed *meandering Miras* by Zijlstra & Bedding (2002) and tend to have nonmonotonic period variations of order 5%–10% of their pulsation period. A smaller fraction still ( $\sim 1\%$ –2%) have exhibited continuously increasing or decreasing periods with changes exceeding  $\sim 10\%$ , and may be in the midst of *thermal pulses* (Wood & Zarro 1981). Large period changes, though rare, have been well documented in a few stars since the earliest

work of Sterne & Campbell (1937), and centuries-long historical light curves for a few Miras show significant changes over time (Sterken et al. 1999; Percy & Au 1999; Zijlstra et al. 2002), even if periods are relatively stable at present. Templeton et al. (2005) used the American Association of Variable Star Observers (AAVSO) data archives to search for large, long-term period changes in 547 Mira variables, including *o* Ceti itself. Although *o* Ceti was not flagged as having a large period change, the significance criteria used in that paper were very high, and did not rule out period changes at the level of a few percent. More subtle period changes including the seemingly random cycle-to-cycle ones may be of astrophysical interest.

*o* Ceti itself may exhibit long-term variations in pulsation behavior as well. Barthes & Mattei (1997) used the visual data from the AAVSO data archives and found long periods (from 600 to over 1500 days) with marginal statistical significance. The detections were not of high statistical significance (primarily due to broadband, low-frequency power), but this raises an interesting question about long periods in *o* Ceti and in the Miras generally. There have been reports of other long-term variations in the Mira AB system in the past (Joy 1954; Yamashita & Maehara 1977; Baize 1980; Karovska 1992; Wood & Karovska 2006).

Long periods and long-period variations are known to occur in AGB pulsators, though the physical processes that generate them are not understood, and there is no conclusive evidence of long multiperiods in Miras. Hinkle et al. (2002) used spectroscopy to study a sample of nine AGB variables with long secondary periods (LSPs), and found six of the nine stars showed radial velocity variations that confirmed the photometric secondary periods but are interpreted as *not* being caused by binarity. The one Mira variable in their sample with a suspected LSP (SV And) showed no secondary radial velocity variations. Wood et al. (2004) also used spectroscopy to study three AGB stars known to have secondary periods in an attempt to determine the origin of this behavior. They found that the data support none

of the suspected origins of these periods (radial or nonradial pulsations, binarity, or stellar activity and spots) unambiguously; those authors suggest that low- $\ell$ ,  $g+$  modes in the radiative portion of the red giant are best supported by the data. Among the Milky Way population, Percy & Bakos (2003) showed that many pulsating red giants (as many as 30% of them) show LSPs on the order of hundreds or thousands of days, in addition to their shorter periods of tens to one hundred days. Recently, the spotlight has been placed on long-term secondary periodicity in semiregular variables and pulsating giants observed in the Large Magellanic Cloud (LMC). Wood et al. (1999) clearly showed that there also exists a well populated sequence of red giant stars in the LMC period–luminosity diagram with secondary periods of several hundreds to 1000 days. In both cases, the causes for these long periods are not clear, but the suggestions of binarity, and nonradial, strange-mode, or dust-driven pulsation were invoked. Long-period variations clearly exist in red giant AGB pulsators, but the physical picture of why they occur is as yet unclear.

We have undertaken a new study of the long-term light curve of *o* Ceti with the aim of exploring the long-term variability of the class prototype of the Mira variables. It is critical, given the huge amount of observational time devoted to this object, that the long-term observational record be sensitively and self-consistently analyzed to assess whether long-term variations truly do occur in *o* Ceti, and how they manifest themselves. In Section 2, we present a newly compiled light curve for *o* Ceti drawn from all available archives of visual observations, and discuss techniques and strategies that we used to search for long-term variations in the data. In Section 3, we present our results, discussing the nature of the observed variabilities and placing statistical limits on the likelihood of coherent secondary periods and long-term variability. In Section 4, we present a discussion of the results.

## 2. DATA AND ANALYSIS

The primary data set of *o* Ceti used in this study is composed entirely of visual magnitude estimates made by several hundred individual observers since 1902. The majority of these data were taken from the AAVSO International Database. The AAVSO International Database already includes observations from the Association Française des Observateurs d’Etoiles Variables (AFOEV). To these data, we added observations not already submitted to the AAVSO archives from three other organizations: the British Astronomical Association Variable Star Section (BAVSS), the Royal Astronomical Society of New Zealand (RASNZ), and the Variable Star Observers League of Japan (VSOLJ). The AAVSO data have been checked for transcription and other errors with a *validation* procedure described in Malatesta et al. (2006). Data from the three other organizations were not validated during the AAVSO data validation project, but were checked for transcription or keypunch errors for the present project, and all duplicate and highly discrepant points (more than 1 mag away from the mean) were removed prior to analysis. The resulting visual light curve covers the time period of JD 2,415,998.5 (1902 September 6) to JD 2,453,883.9 (2006 May 28), and is the first light curve assembled by combining data from all major variable star data archives worldwide.

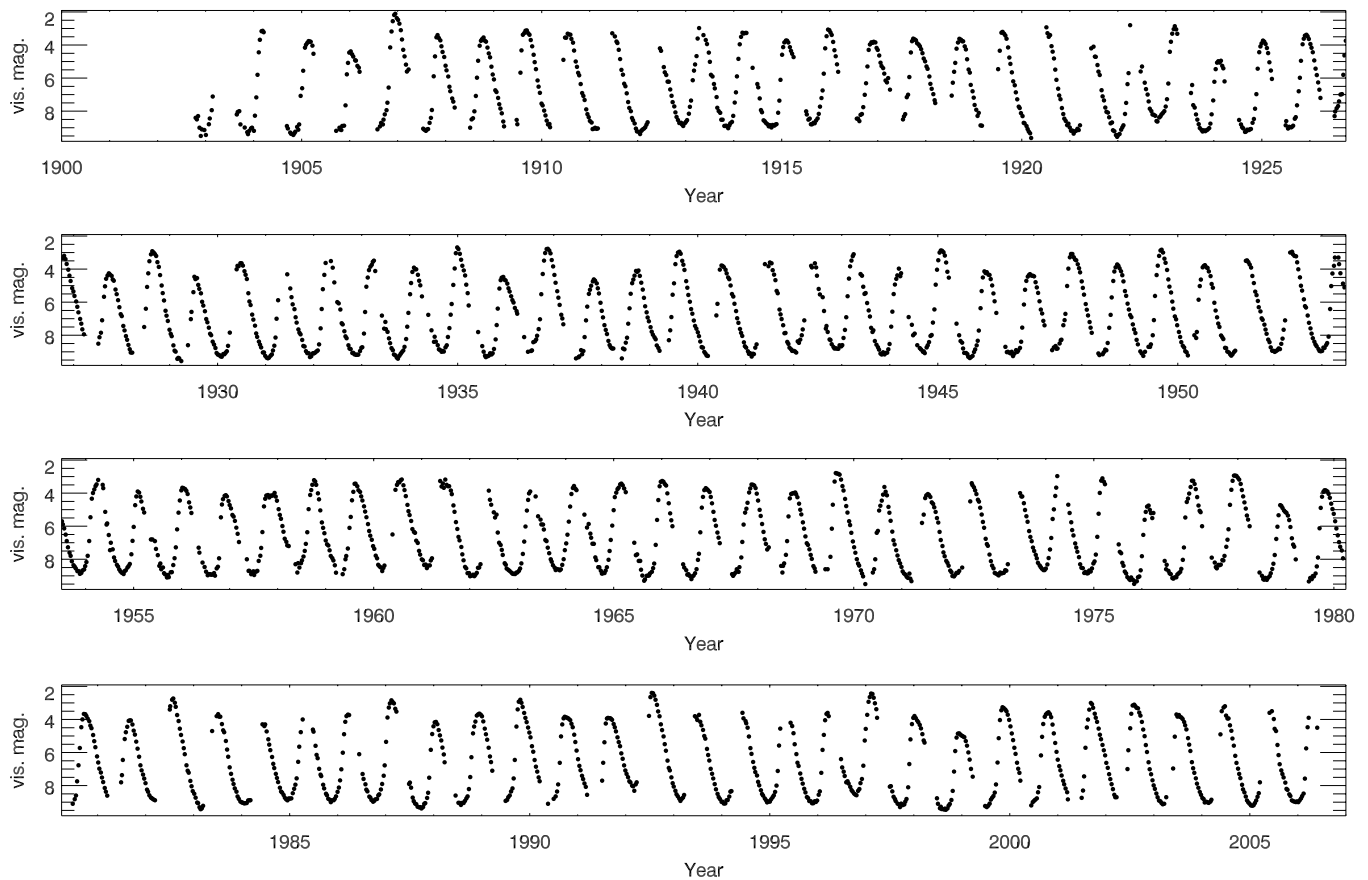
We formed 10 day averages of the visual observations to reduce the scatter from both random errors and systematic differences between observers, and to ensure that the analyses are not temporally biased toward the better sampled parts of the

light curve. Averaging increases the signal-to-noise ratio of the visual data at the minor cost of reduced temporal sensitivity; since all periods of interest are larger than 20 days, the 10 day averaging has minimal impact on our analysis. To assess the internal photometric consistency of the data, we measured the standard deviation of the visual magnitude estimates in each 10 day bin; this quantity averaged 0.2 mag at both maximum and minimum, and about 0.4 mag during the rise and decline phases. The lower scatter during maximum and minimum is due to the historically intensive coverage of *o* Ceti during maximum and minimum phases, and due to the fact that the star changes more over a 10 day span during the rise and decay phases than during maximum and minimum. The magnitudes of scatter during maximum and minimum are essentially identical, despite the factor of a few hundred difference in the flux between the two phases. This is due both to the quality of the comparison star sequence for *o* Ceti, and to the fact that the star is typically observed with larger-aperture telescopes at minimum compared to maximum.

The light curve shown in Figure 1 contains all available data from the five major variable star observing organizations, and is likely as complete a record of the last century’s history of *o* Ceti as has ever been compiled. Every cycle since 1902 has partial or complete coverage, with the only unavoidable exceptions being the annual gaps due to solar conjunction. Many time-series analysis techniques are sensitive to gaps in data and so our main motivation in compiling these data was the minimization of this source of interference. For our study, we focus our analysis only on these 105 years of data. There exist significant data prior to 1902, with good (but not complete) coverage extending to the late 18th century, and very early observations extending to Mira A’s discovery in 1596. This longer light curve was recently collected from literature sources by E. Zsoldos & G. Marschalló and analyzed for periodic amplitude variations (Marschalló 2004). They have kindly provided their light curve to us and we also analyze these data for LSPs and period variations as a check of our primary analysis.

The photometric behavior of *o* Ceti is complex, as it is in nearly all Mira variables, and so we performed time-series analyses of the data using several different techniques. The use of multiple analysis techniques serves both to more fully understand the behavior of the data and to provide a consistency check, since all techniques should yield similar results to within the statistical limitations of each. We used three different classes of tests: Fourier-based time-series analyses, phase-dispersion minimization (PDM), and a Box–Jenkins autocorrelation model.

Fourier-type algorithms used in this work include the date-compensated discrete Fourier transform (DCDFT) as derived by Ferraz-Mello (1981) and implemented by Foster (1995); the Roberts et al. (1987) *clean* algorithm for which we developed our own implementation; and the weighted wavelet Z transform, a time–frequency analysis method developed and implemented by Foster (1996). All of these methods share a fundamental reliance on Fourier signal modeling, but have different approaches to the problem. The DCDFT is essentially a least-squares model using periodic basis functions to find the best-fitting period, amplitude, and phase; the times of the data points are explicitly taken into account to allow uneven sampling. The *clean* algorithm uses iterative peak finding and subtraction using the data window function to separate real signals in the Fourier transform from spurious alias peaks caused by the annual data gaps. Finally, the wavelet Z algorithm uses a standard Fourier fitting algorithm but with the novelty of a sliding Gaussian weighting



**Figure 1.** Light curve of *o* Ceti, from 1902 to 2006. Each point is the average of 10 days' worth of visual observations.

window to explore the change in the data's signal content as a function of time.

Two other statistical tests were used. First, we used the PDM algorithm of Stellingwerf (1978) to test for long-term periodic behavior. PDM folds the data on a set of test periods, and bins the data for each test period to find that with the lowest dispersion per bin, a sign of periodic behavior with that period. Second, we used an autoregressive integrated moving average (ARIMA) model with a seasonal adjustment (i.e., a SARIMA model) of the dominant pulsation period. A Box & Jenkins (1976) autocorrelation of the adjusted data was then done to test whether there were any other correlated timescales in the data.

### 3. RESULTS

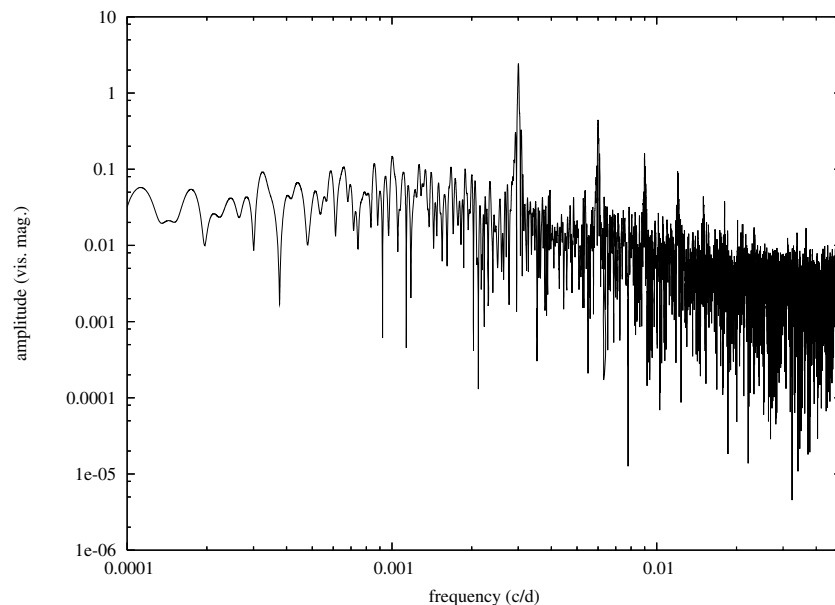
We sought to answer two questions about the long-term behavior of *o* Ceti with our analyses: (1) how stable is the pulsation period over time and (2) are there stable, LSPs present in the light curve? The former question is important because Miras are known to have strong cycle-to-cycle variations in their light curves which statistically manifest themselves as changes in period or amplitude, and it is important to determine whether observed period changes are simply due to random processes or something else like evolutionary or other long-term internal structural changes. The latter question is clearly also important because secondary periods are indicators of periodic processes in or around the star, either internal pulsations or external modulations such as interactions with a companion object. The different time-series techniques described in the preceding section can provide answers to different questions depending upon the type of analysis. Fourier and folding

methods can most easily detect stable periods, but also provide hints as to the stability of known periods. Wavelet or time-frequency methods like the wavelet Z transform can show the time evolution of the spectrum, since it essentially performs a localized Fourier analysis of short sections of the light curve. Finally, autocorrelation can help to uncover behavior which may not be strictly periodic, or which is not present or in phase throughout the entire light curve.

During the course of our analysis, it became clear that one of the features of the variability was a low-frequency power-law component, qualitatively similar to that observed in other giant and supergiant pulsators by Kiss et al. (2006). A third test that we performed was to quantify the nature of this power law, and determine whether it quantitatively matched what was observed in the red supergiants. We discuss each of these three issues in the following subsections.

#### 3.1. Pulsation Period and Period Stability of *o* Ceti

All time-series analysis methods yield the same dominant period of variability to within the calculational uncertainties of each; the strongest period derived from the *clean* analysis is  $333.09 \pm 0.04$  days (see Figure 2). All integer harmonics of this period through  $P_0/6$  ( $= 55.52$  days) also appear in the Fourier spectrum, as do the lunar sidereal and synodic periods, caused by a weak modulation in sky brightness during the lunar cycle. It is also apparent that the dominant pulsation period is modulated, as the spectral peak is actually a blend of multiple peaks, as are those of the Fourier harmonics; all of the multiple peaks are contained within Lorentz- or Gaussian-shaped envelopes centered on the main period as was noted by Kiss et al. (2006).



**Figure 2.** Cleaned Fourier transform of the light curve of  $\alpha$  Ceti showing the dominant frequency of  $0.003\text{c/d}$  ( $P = 333$  days) and its higher-frequency harmonics. The spectrum is featureless other than the low-frequency power-law continuum, and very weak peaks ( $< 0.05$  mag) at the lunar synodic and sidereal periods.

We used the Period04 (Lenz & Breger 2005) package to fit and prewhiten the data with the periods  $P_0$  through  $P_0/6$  noted above and found that subsequent peaks could not be reliably fitted with any set of periods; allowing the periods of  $P_0$  and its harmonics to vary along with those of additional peaks resulted in divergent solutions unless unphysically large numbers of periods were used in the fit. This indicates that the pulsation is changing with time; it is not clear whether the period, amplitude, phase, or a combination of the three are changing, but the times between individual light curve maxima and minima are not constant to better than 4–5 days from cycle to cycle.

We used the weighted wavelet Z transform (Foster 1996) to explore how the pulsations varied with time, and whether the period was being coherently modulated on a timescale measurable with the 105 year light curve. Figure 3 shows the peak period of  $\alpha$  Ceti as measured with several different window widths of the wavelet transform; the parameter  $c$  is equal to  $(2\pi n_c)^{-2}$ , where  $n_c$  is the  $e$ -folding half-width of the weighting window in cycles. As the width of the window is increased, the variations in period centroid become much less erratic, suggesting that cycle-to-cycle variations in the period and/or the noise of the data are large. It is difficult to analytically determine the errors on Foster’s wavelet Z statistic, but a reasonable estimate of the *maximum* error is the confusion limit of the spectrum, which can be measured as the half-width of the wavelet Z spectrum at each lag time. For values of  $c > 0.001$ , the half-width of the peaks is much greater than 5 days, which is more than the variation of the period maximum itself. For values of  $c = 0.00025$  and  $0.001$ , the half-widths are on the order of the observed variation. Since this value is the confusion limit rather than the error in period due to noise or other considerations, the actual error bars are likely smaller. Based on this, there appear to be marginally significant period changes in  $\alpha$  Ceti at a level of 4–5 days over the course of the past 105 years. Although the span of the data is short relative to the timescales in question, there appears to be both a trend toward increasing average period, and a marginally significant oscillation in period with a timescale of  $\sim 30$ – $35$  years, and amplitude of 1–2 days. This is a marginal detection, but the increasing period is consistent with

the expected evolutionary trend, and the oscillatory behavior is strikingly similar to that observed in the *meandering Miras* (Zijlstra & Bedding 2002), albeit with much lower amplitude.

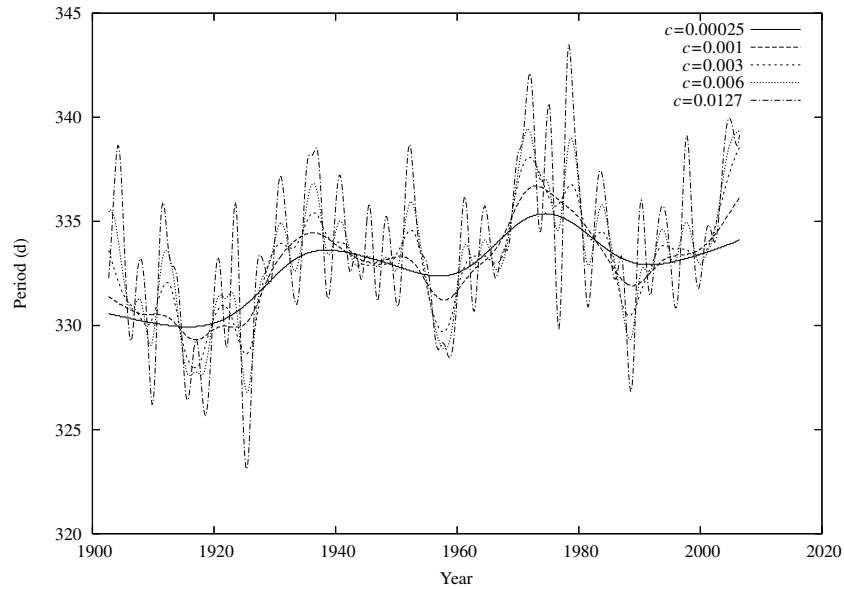
### 3.2. Multiperiodicity

LSPs have been found in large numbers of long-period semiregular variables in some surveys (Derekas et al. 2006; Soszyński 2007; Fraser et al. 2008), but as was mentioned in the Introduction, the causes of these periods are generally unknown and they have not been conclusively detected in Miras. Barthes & Mattei (1997) noted that several long periods were present in the  $\alpha$  Ceti light curve with marginal statistical significance, and one motivator for our project was to investigate these claims with a longer and more complete data set than that used in the earlier analysis.

When the entire 105 year light curve was analyzed together, no coherent LSPs were detected in the data with any statistical significance. This suggests that such periods are not present, that they are not present throughout the entire light curve, or that they are at low amplitude. To test for transient LSPs, we subdivided the 105 year light curve into 6000 day segments, the length of which was chosen arbitrarily to cover more than a dozen cycles of the dominant variation, and which could in principle detect periods as long as 3000 days. The light curve was split so that consecutive segments overlapped by 3000 days; any given point in the light curve appears in two adjoining segments, but every other segment represents a wholly independent data set. All data segments were again analyzed with the cleaning Fourier transform, and again no LSPs having periods shorter than 3000 days were found with statistically significant amplitude. It is possible that there could be transient periods longer than 3000 days in the full light curve, but there was no evidence for this in the analysis of the full data set.

Finally, to search for any kind of quasiperiodic behavior or variations with a characteristic timescale, we employed an autoregressive moving average (ARMA) model to the data, followed by an autocorrelation analysis as devised by Box & Jenkins (1976). This particular model includes a seasonal





**Figure 3.** Variation of the dominant period's centroid as a function of time, measured with the weighted wavelet Z time–frequency algorithm (Foster 1996). Results are shown for several different widths of the weighting window, ranging from  $c = 0.00025$  to  $0.0127$ . For larger values of the window width parameter,  $c$ , the uncertainty in the period determination is much larger than the variation, but at the lowest values of  $c$  the variation in period is on the order of the *maximum* uncertainty in the period.

adjustment (known as a SARIMA model), since it is already known that the data contain a strong period of 333 days. The raw visual observations were rebinned into one-day averages, and the data were resampled onto an even grid using the algorithm of Reinsch (1967). We then formed a new set of data by subtracting both the value of the nearest-neighbor magnitude, as well as that of the data point 333 days prior and its nearest neighbor:

$$W_i = (m_i - m_{i-1}) - (m_{i-333} - m_{i-334}). \quad (1)$$

This subtraction is completely analogous to the typical monthly or seasonal adjustment (e.g.,  $X_i - X_{i-12}$ ) often seen in similar autocorrelation analyses of terrestrial or socioeconomic data. The new data set  $(t_i, W_i)$  was then analyzed with a Box & Jenkins (1976) autocorrelation algorithm. A plot of the autocorrelation parameter  $r(k)$  versus lag time for the adjusted data is shown in Figure 4. The adjusted data are essentially uncorrelated at any period, except for an anticorrelation at the known period of the pulsation. The lack of correlation peaks on any timescale up to half the length of the light curve strongly suggests no periodic or quasiperiodic variability is present in *o* Ceti.

As we will show in Section 3.4, the spectrum contains a strong red noise component, with higher amplitude fluctuations at longer periods. This suggests that the peaks observed by Barthes & Mattei (1997) may have been these stochastically generated fluctuations rather than the discrete modes mentioned as a possible cause. The amplitude of the background has a  $1/f^\alpha$  shape, with an amplitude of roughly 0.1 mag at a period of 1000 days. We can rule out the presence of LSPs in *o* Ceti with amplitudes greater than this. We cannot rule out very low amplitude pulsations, but the stars exhibiting LSPs generally have amplitudes larger than this limit.

### 3.3. Analysis of the 170 year Light Curve

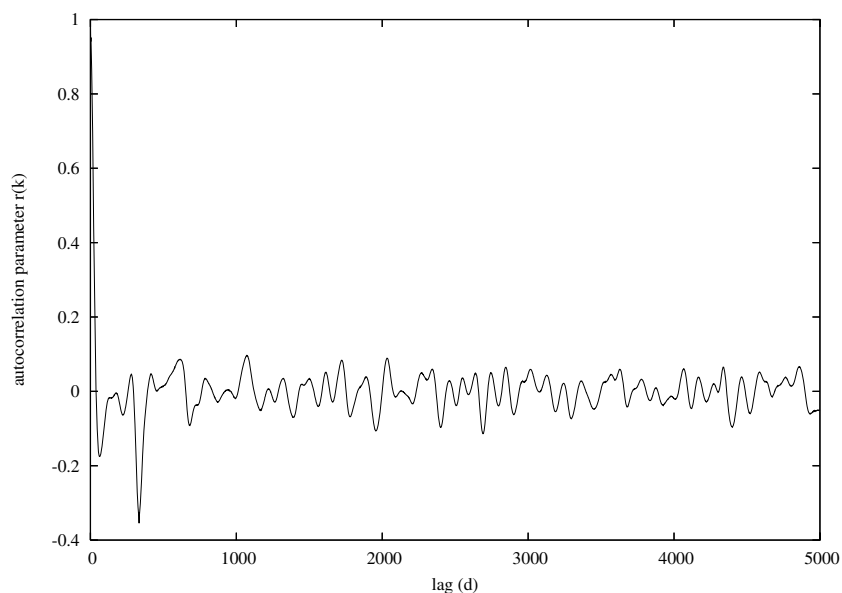
We performed some of the above analyses on a subset of the light curve kindly provided by E. Zsoldos and G. Marschalló (Marschalló 2004) to determine whether any periodic behavior

might be present during earlier times, or whether the use of a longer set of data might raise the signal-to-noise ratio of low-amplitude variability in the spectrum. The light curve provided by Marschalló contains data as far back as the 17th century, but for the purposes of time-series analysis, only the data beginning with the observations of F. Argelander in the 1830s are well sampled enough to provide reasonable coverage for our purposes, and we used only data beginning at JD 2392722 (1838 December 13) up through the start of our own data set at JD 2416000. These data were combined, and again averaged into 10 day wide bins. The light curve of the early data is shown in Figure 5.

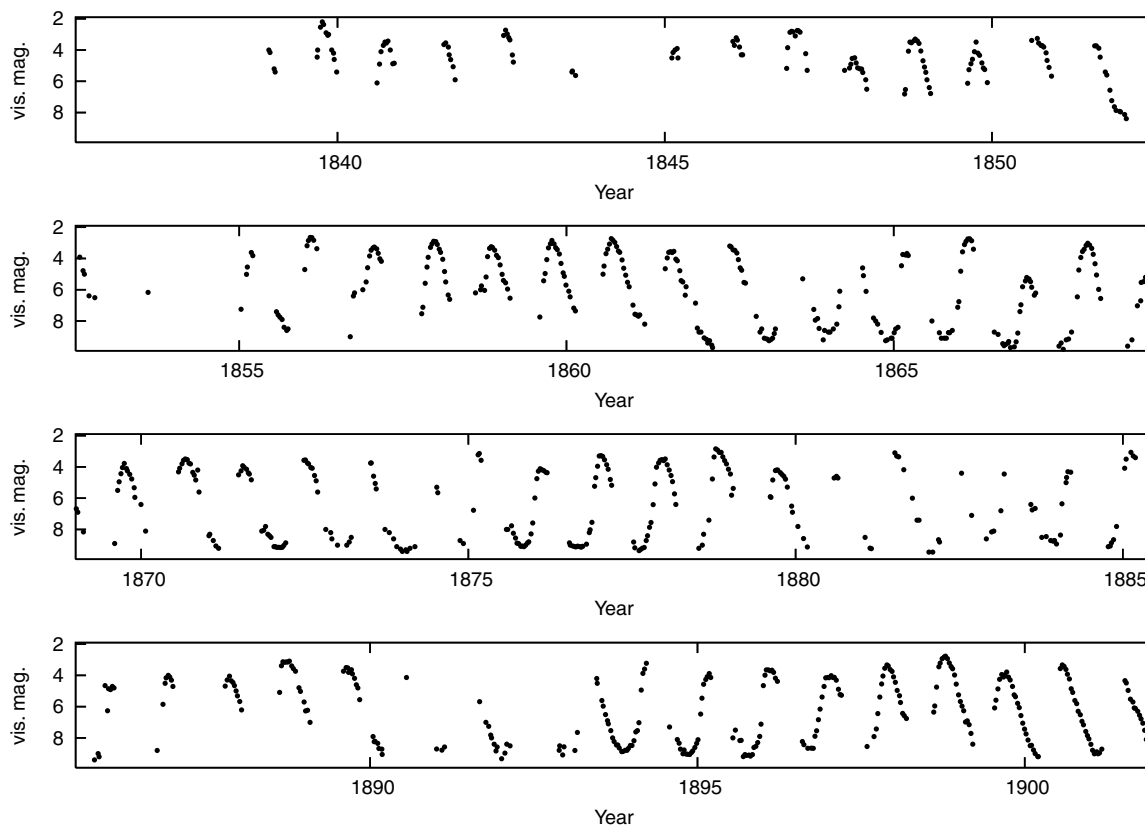
When all of the data from JD 2392722 to the present are analyzed, the resulting spectrum (Figure 6) looks very similar to that of the light curve beginning on JD 2,416,000 as is expected. There is no clear evidence of LSPs, and although the low-frequency noise structure is similar to that of the transform of the shorter light curve, the peaks do not always match in frequency. Thus, we do not believe there are weak LSPs present in the longer light curve. We then subdivided the data into 6000 day segments as was done for the shorter light curve, and again found the same variation in period (Figure 7), with no evidence of transient LSPs. We note that the variations in period—both the period increase and the decades long modulation in period—appear to extend back to the earliest portion of the light curve. However, we decided to discard the first six thousand days of data (up to JD 2,398,000) because the amplitudes returned for the dominant period were so discrepant that we believe the transform was corrupted by the lack of observations near *o* Ceti's minimum. The two data points thus discarded do support the period modulation, but not the long-term trend.

### 3.4. The Low-Frequency Power-Law Component

All of our time-series analyses indicated an excess of noise power at low frequencies, indicative of an underlying red noise component to the variation which is common in chaotic or



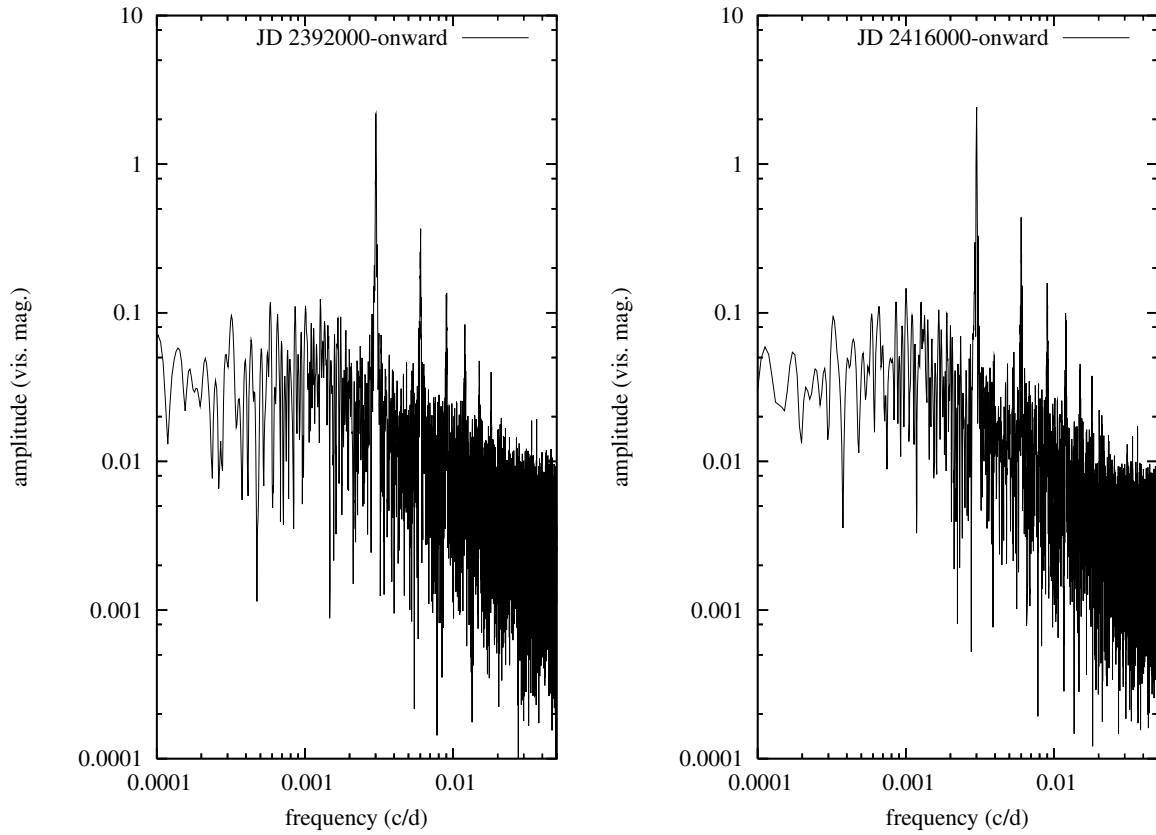
**Figure 4.** Autocorrelation parameter  $r(k)$  of the  $\alpha$  Ceti data fitted with a SARIMA model. No significant correlations appear in the data following the trend and seasonal adjustments, indicating there are no coherent LSPs in the data.



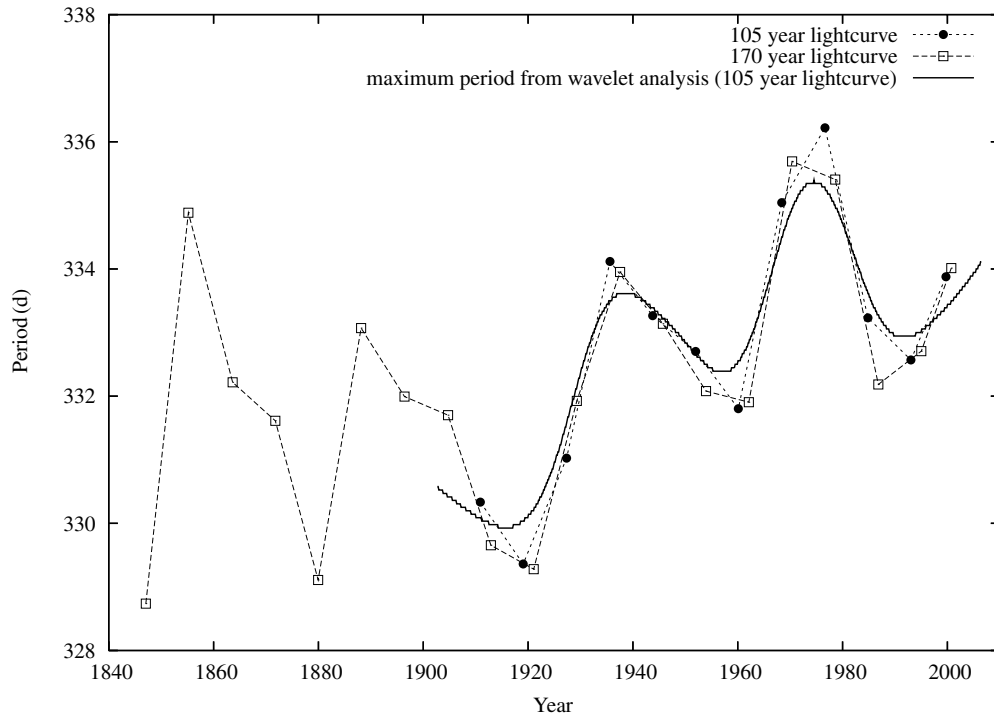
**Figure 5.** Light curve of  $\alpha$  Ceti, 1838–1902. These data are much more sparsely covered than the 102 year light curve shown in Figure 1, although the overall behavior is clearly apparent. Digitized data kindly provided by E. Zsoldos & G. Marschalló.

quasiperiodic systems. To investigate this feature, we followed the analysis of Kiss et al. (2006) to measure the spectral index of the spectrum. We removed six bands from the spectrum directly rather than prewhitening the data and Fourier transforming the residuals because the pulsation periods are not stable enough for effective prewhitening of such high amplitudes. The bands subtracted were defined by  $i(f \pm \delta f)$ , where  $i = 1, 6$ ,  $f = 0.003c/d$ , and  $\delta f = 0.0002$ . Following Kiss et al. (2006),

the spectrum was then averaged into logarithmic frequency bins 0.1 dex in size; because the pre-filtering removed large blocks of frequencies, the frequency of any given bin was the average of the frequencies falling in that bin rather than the bin center. The averaged power per bin was then normalized with the length of the data set to calculate the power density. We also performed the same test on the longer, 170 year light curve. The spectra of power density versus frequency for both light curves



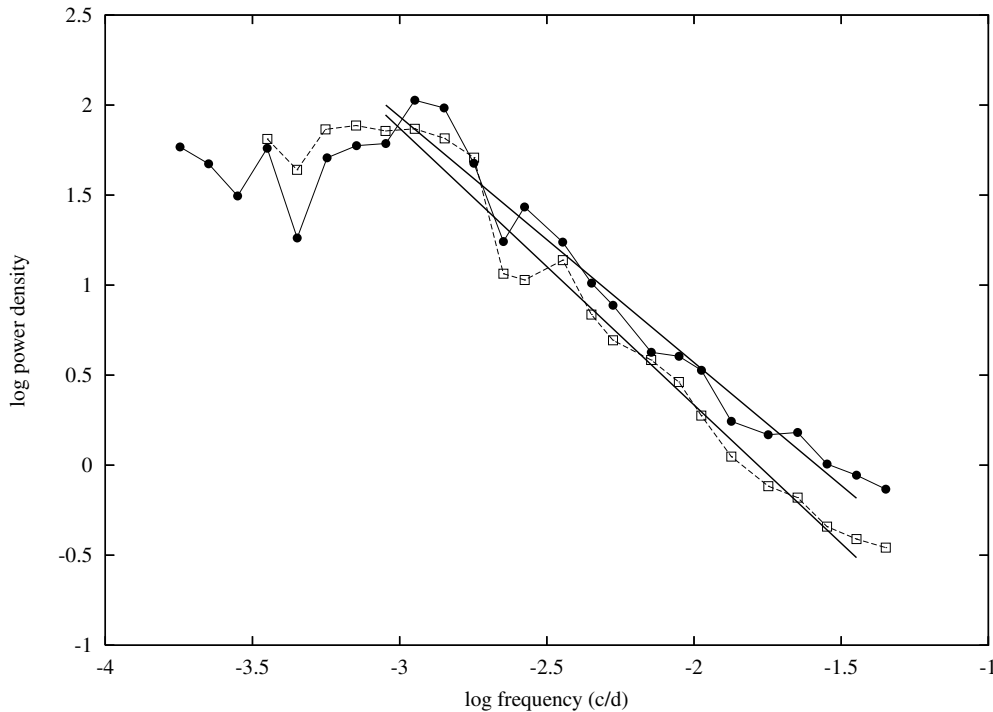
**Figure 6.** Cleaned Fourier transforms of 105 year (right) and 170 year (left) light curves. Both light curves show the same overall features including dominant frequencies and their amplitudes, as well as slope of the power law, and flattening of the spectrum for frequencies below  $0.001\text{c/d}$ .



**Figure 7.** Period change vs. time as measured using 6000 day segments (points) and wavelet analysis (solid line). The centroid periods of the cleaned spectra match those of the wavelet analysis, indicating that the wavelet analysis is correctly measuring the correct period. The oscillatory nature of the main period variation continues in the longer, 170 year data set for data prior to 1900, suggesting that it is real.

are shown in Figure 8, along with lines indicating the best-fit power laws for each. For the 105 year light curve, the best-fit

power law has  $\alpha = -1.29$ , while for the 170 year light curve, the best-fit power law has  $\alpha = -0.86$ . These two are different,



**Figure 8.** Power density spectra of 105 year (open squares) and 170 year (filled circles) light curves of *o* Ceti, showing the best-fitting power laws for  $-3 < \log f < -1.4$ . The values of the power-law slopes are  $\alpha = -1.54$  for the 105 year light curve, and  $\alpha = -1.37$  for the 170 year light curve, which are in reasonable agreement. Both spectra also show a turnover at  $\log f < -3$ , which suggests it is a real effect, rather than due to the finite length of the shorter light curve.

but this has not taken into account the fact that the slope of the 170 year data set includes more low-frequency data lying within the turnover region. When power laws are fitted to data only between  $\log f = -3$  and  $\log f = -1.4$ , the slopes for the 105 and 170 year light curves are  $\alpha = -1.54$  and  $\alpha = -1.37$ , respectively, which agree within the uncertainties.

There is ambiguity in the nature of the power law which leads to ambiguity in the physical explanation for the behavior. A slope  $\alpha = -1$  can approximate the slope over the entire range of the spectrum, but would indicate a power excess at  $\log f \sim -3$ . A steeper slope of  $\alpha = -1.4$  would fit the spectrum between  $-3 \leq \log f \leq -1.3$ , but would indicate a strong turnover for  $\log f < -3$ . Aside from the slight depression in power around  $\log f \sim -2.5$  ( $P \sim 333$  days), the power density spectrum is a smooth continuum between  $\log f \sim -3$  and  $-1.5$ , with a best-fit power law having a slope of  $\sim -1.37$ , and a strong turnover for  $\log f < -3$ . This could be interpreted as a cutoff in power for low-frequency variability, either caused by the lack of variations on very long timescales, or a maximum amplitude for such variations. If instead a single power law were used for the entire spectrum, then there is evidence for a bump in power with periods around 1000 days. We do not see coherent variations with those periods either in the full data set or in the 6000 day segments, so we do not believe these could be unresolved long-period modes.

#### 4. DISCUSSION

Our analysis of the visual light curve of *o* Ceti has shown three things: that the only definite *periodic* signal is that of the pulsation of *o* Ceti, that there is a marginally significant systematic variation in period of approximately  $\pm 2$  days over the past century, and that there appears to be a low-frequency power-law component to the Fourier spectrum. The use of a

spectral cleaning algorithm that removes the effects of the window function substantially reduced the number of peaks in the final spectrum, and what remained appears to be either directly related to the pulsation period and its integer harmonics, or to the underlying stochastic power law. We set out to discover at the start of this project whether *o* Ceti was among the AGB stars with LSPs, and we have definitively ruled this out at the level of 0.05–0.1 mag for periods between 300 and a few thousand days.

On the other hand, the computed power spectra of *o* Ceti—both for the entire light curve and for the 3000 day data segments—all show very strong excess of power at low frequencies. The physical cause of such a power-law component is likely the brightness variations across the stellar surface generated by convection. The recent work of Kiss et al. (2006) clearly showed that convection is strong in red supergiants, and the effect would be similar if diminished for the smaller AGB stars. We believe that the low-frequency spectrum in *o* Ceti is generated by the same processes that cause the low-frequency power law in the red supergiants.

For example, Schwarzschild (1975) predicted that convective cells with sizes on the order of 1/4 to 1/30 of the visible portion of the stellar disk may generate detectable photometric variations with timescales on the order of the survival lifetime of the cell, namely a few to several hundred days. Likewise, Antia et al. (1984) predict that large convective cells in red giant envelopes appear to be those preferentially selected. The appearance, evolution, and destruction of these convective cells (with the concurrent changes in local surface temperature, surface brightness, and possibly dust opacity) would result in photometric variability. If the cells are generated with a range of sizes as happens in stochastic processes like convection, it is reasonable that the resulting photometric variability would have a power-law spectrum as well.



In addition, we detect in *o* Ceti a weak feature showing an apparent flattening of the power law for periods longer than approximately 1000 days. The reality of this turnover and its cause are not certain, and it may be due to the finite length of the light curve. However, the presence of the turnover in the much longer light curve including archival data from the mid-19th century argues that the turnover is real. One possible origin for a turnover is that the strong pulsation in *o* Ceti may cause dissipation of convective cells with turnover times longer than the pulsation period of 333 days. Another possibility is that the size of *o* Ceti itself, smaller than the red supergiants, may place an upper limit on the sizes of convective cells that can form to those with turnover times less than several hundred days. Our results for *o* Ceti and the results for the red supergiants obtained by Kiss et al. (2006) highlight the necessity of high-angular resolution observations of *o* Ceti and other Mira variables with the capability of imaging the stellar surface variations.

## 5. CONCLUSIONS

Our time-series analysis of the longest continuous light curve of *o* Ceti reveals no long-term, coherent periodic behavior at any period, to a significance level of approximately 0.1 mag at  $P = 1000$  days. Our analysis has revealed the presence of a low-frequency power-law component which can generate quasi-periodic, low-frequency variability on short timescales. It is this low-frequency power-law component that is responsible for the upper limit of 0.1 mag, not the photometric precision of the visual data; if there is an LSP present, it must have an amplitude lower than that of the low-frequency stochastic variability. We would have detected any LSP with an amplitude greater than 0.05 if it were not overwhelmed by the intrinsic noise. The physical origin of the stochastic variability is unknown, but it may be a signature of photometric variability caused by convective supergranulation on large timescales first suggested by Schwarzschild (1975).

We have detected a modulation in the dominant pulsation period of *o* Ceti with a timescale of approximately 30 years. This modulation is detected with marginal statistical significance, but the timescale matches other observed modulations in the physical behavior of the Mira AB system (Baize 1980; Karovska 1992).

We have shown that using long-term visual light curves of Miras we can explore low-frequency variability to amplitude limits below 0.1 mag. A number of Mira variables have very long light curves, and in principle periods of 20 times the dominant pulsation period could be detected in many of them. A large-scale population study of pulsating AGB stars looking specifically for LSPs and correlating them with spectral type and other stellar properties is warranted. Our understanding of these objects would be improved by careful scrutiny and analysis of the light curves themselves.

We wish to emphasize in closing that the significance limits on large ensembles of visual data (for example the long-term Mira light curves) are clearly better than the 0.2–0.3 mag *per estimate* commonly quoted. When a large set of data is used in a study such as this, it is very straightforward to detect periodic behavior at amplitudes on the order of 0.05 mag or less. Our nondetection of coherent, long-period variability in *o* Ceti itself is not a limitation of the data quality, but is caused by the presence of an intrinsic low-frequency power-law component which would overwhelm any existing coherent signal. Visual observations in

sufficient quantity may certainly provide much more sensitive measures of stellar variability than is assumed, and long spans of visual observations should be used to perform similar studies with other bright and well observed Mira variables.

We thank the many thousands of observers around the world who have contributed observations of this and all other variable stars to the AAVSO and all other variable star organizations worldwide. We extend our special thanks to R. Pickard of the BAAVSS and R. MacIntosh of the RASNZ for personally supplying the data archives of those two organizations, and to the AFOEV and VSOLJ for making their archives available. We thank E. Zsoldos & G. Marschalló for supplying an electronic copy of their Mira light curve. We also thank the anonymous referee whose comments substantially improved this paper. M.K. is a member of the *Chandra* X-ray Center, which is operated by the Smithsonian Astrophysical Observatory under NASA Contract NAS8-03060.

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